Quantum Measurement Information as a key to Energy Extraction from Local Vacuums

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Abstract

In this paper, a protocol is proposed in which energy extraction from local vacuum states is possible by using quantum measurement information for the vacuum state of quantum fields. In the protocol, Alice, who stays at a spatial point, excites the ground state of the fields by a local measurement. Consequently, wavepackects generated by A' measurement propagate the vacuum to spatial infinity. Let us assume that Bob stays away from Alice and fails to catch the excitation energy when the wavepackets pass in front of him. Next Alice announces her local measurement result to Bob by classical communication. Bob performs a local unitary operation depending on the measurement result. In this process, positive energy is released from the fields to Bob's apparatus of the unitary operation. In the field systems, wavepackets are generated with negative energy around Bob's location. Soon afterwards, the negative -energy wavepackets begin to chase after the positive-energy wavepackets generated by Alice and form loosely bound states.

1 Introduction

The interplay between quantum information theory and quantum field theory has intensified and is expected to revolutionize physics. For example, novel ideas are proposed on, for example, the information loss problem of black holes[1] and quantum causal histories of quantum gravity[2]. It has been also discussed that distillation of vacuum-entanglement of quantum fields yields EPR pairs [3] and W states [4]. In reference [5], the method of field theory in curved spacetime is employed to evaluate actuating energy of photon switching in quantum communication. In this paper, we apply positive operator value measure (POVM) and local operations and classical communication (LOCC) to the physics of negative energy density in quantum field theory. POVM and LOCC are fundamental tools of quantum information theory[6].

Quantum fluctuations of the local energy around its zero value in field theory has been studied for a long time [7]. Quantum interference is able to create states containing regions of negative energy, though the total energy remains nonnegative. A notion of negative energy has impacts on many fundamental problems of physics, including traversable wormhole [8], cosmic censorship [9] and the second law of thermodynamics [10]. It has been pointed out that available absolute values of negative energy are crucial for those problems. Possible values of negative energy are restricted by quantum inequalities for energy density based on uncertainty relations [11][12].

Classical energy of free fields takes nonnegative values, and cannot be used successfully when our apparatus of energy extraction from the fields is located outside nonvanishing-energy regions. This situation is dramatically different for quantum energy. Let us consider a local quantum measurement performed for the vacuum state. A finite amount of positive energy is infused into the system at the measurement device position. Because the properties of states excited by local measurement are the same as those of the vacuum outside the excited regions, those states can be called local vacuum states. The concept of local vacuum states is the same of that of strictly localized states proposed by Knight [13]. In this paper, it is proven for a free massless scalar field in 1+1 dimensions that the excitation energy can be partly extracted back from the field using the measurement results and a quantum apparatus located away from the measurement point, *even if the field has, on average,*

no energy around the apparatus at all. Using this method, we can transport energy to a distant location by sending not a physical object with excitation energy, but classical information. In the extraction process, wavepackets with negative energy density are generated in the system and form loosely bound states with positive-energy wavepackets excited first by the measurement device. This method is based on a quantum energy teleportation protocol proposed for spin chains [14]. The protocol transfers localized energy from one site of a spin chain to another only by LOCC. However, aims of the paper are confined to short-time-scale processes in which dynamical evolution induced by the Hamiltonian is negligible, although LOCC is assumed possible many times in the short interval. In relativistic field systems, the dynamical effect propagates with light velocity, which is the upper bound on the speed of classical communication. Thus, we generally cannot omit global time evolution. It is also noted that any continuous limit of zero lattice spacing cannot be taken for the protocol in [14] because measurements in the protocol are projective, which becomes an obstacle to obtaining a smooth limit. In this paper, we adopt a different general measurement that is well defined in field theory.

The paper is organized as follows. In section 2, we briefly review general measurements and LOCC of quantum information theory. In section 3, a short review of negative-energy physics of a 1+1 dimensional free scalar field is given. Section 4 presents a protocol in which energy is extracted from local vacuum states using measurement results. In section 5, an explicit example of the protocol is given. Section 6 summarizes the results. We adopt the natural unit$c = \hbar = 1$.

2 POVM and LOCC

In this section, we give an overview of concepts related to general measurements by use of POVM and measurement operator and LOCC in quantum information theory. A detailed explanation can be found in standard textbooks of quantum information[6]. measurements are generalized measurements beyond projective (ideal) measurements. Let us consider a quantum system S about which we wish to obtain information. In order to formulate general measurements, we need another quantum system S' as a probe. In

general, dimension N of the Hilbert space of S is not equal to that of S'. We bring S' into contact with S by probe interactions between the two. In this process, information on S is imprinted into S'. After switch-off of the measurement interactions, we perform a projective measurement on not S but the probe system S' and obtain imprinted information about S. This completes a general measurement. An ideal measurement can be made if a composite quantum state after switch-off of the interaction is given by

$$|\Psi\rangle_{SS'} = \sum_{n=1}^{N} c_n |n\rangle_S |u_n\rangle_{S'},$$

where $\{|n\rangle | n = 1 \sim N\}$ is the complete set of orthonormal basis state vectors of S and $\{|u_n\rangle_{S'}\}$ is the set of orthonormal state vectors of S'. When a measurement result for S' is given by $|u_n\rangle_{S'}$ with probability $p_n = |c_n|^2$, we infer that S is also observed in the state $|n\rangle_S$ with the same probability. Hereafter we will express quantum states by density operators. General measurements are mathematically described using measurement operators M_μ ($\mu = 1 \sim m$), which act on the Hilbert space of S and satisfy

$$\sum_{\mu=1}^{m} M_\mu^\dagger M_\mu = I_S, \tag{1}$$

where the number of M_μ is denoted by m and generally not equal to N. Let us consider explicitly an indirect measurement model in order to understand the measurement operators. Let us write down a probe Hamiltonian as

$$H_p(t) = \sum_\gamma g_\gamma(t) O_S^{(\gamma)} \otimes O_{S'}^{(\gamma)},$$

where $O_S^{(\gamma)}, O_{S'}^{(\gamma)}$ are Hermitian operators acting on the Hilbert spaces of S and S', and $g_\gamma(t)$ are real functions of time t which take zero values for $t \notin (0, T)$. The interaction generates entanglement between S and S'. The time evolution is described by the following unitary operator.

$$U_p(T) = \mathrm{T} \exp\left[-i \int_0^T H_p(t) dt\right] = \exp\left[-i \sum_\gamma \int_0^T g_\gamma(t) dt O_S^{(\gamma)} \otimes O_{S'}^{(\gamma)}\right].$$

Let us set the initial state as $|\psi_S\rangle$ for S, and $|0_{S'}\rangle$ for S' at $t = 0$. After switch off of the probe interaction, the total state is given by

$$|\Phi\rangle = U_p(T)\left(|\psi_S\rangle \otimes |0_{S'}\rangle\right).$$

Now let us perform a S' projective measurement for $|\Phi\rangle$. Consider a complete orthonormal basis $\{|\mu, S'\rangle | \mu = 1, \cdots, m\}$of the Hilbert space of S'. The index μ classifies m possible outputs of the measurement. The projection operator onto $|\mu, S'\rangle$ is defined by

$$P_\mu(S') = |\mu, S'\rangle\langle\mu, S'|.$$

Because of completeness, the following relation is satisfied.

$$\sum_{\mu=1}^{m} P_\mu(S') = I_{S'}. \tag{2}$$

The measurement operator M_μ is obtained by acting $I_S \otimes P_\mu(S')$ on $|\Phi\rangle$ such that

$$\left(I_S \otimes P_\mu(S')\right)|\Phi\rangle = M_\mu|\psi_S\rangle \otimes |\mu, S'\rangle.$$

It is noted that M_μ are operators acting on the Hilbert space of S. The explicit form of M_μ is given by

$$M_\mu = \langle\mu, S'|U_p(T)|0_{S'}\rangle.$$

Eq.(1) then is easily verified as follows.

$$\begin{aligned}
\sum_{\mu=1}^{m} M_\mu^\dagger M_\mu &= \sum_{\mu=1}^{m}\langle 0_{S'}|U_p^\dagger(T)|\mu, S'\rangle\langle\mu, S'|U_p(T)|0_{S'}\rangle \\
&= \sum_{\mu=1}^{m}\langle 0_{S'}|U_p^\dagger(T)\left(I_S \otimes |\mu, S'\rangle\langle\mu, S'|\right)U_p(T)|0_{S'}\rangle \\
&= \langle 0_{S'}|U_p^\dagger(T)\left(I_S \otimes \sum_{\mu=1}^{m} P_\mu(S')\right)U_p(T)|0_{S'}\rangle \\
&= \langle 0_{S'}|U_p^\dagger(T)U_p(T)|0_{S'}\rangle \\
&= \langle 0_{S'}|I_S \otimes I_{S'}|0_{S'}\rangle = I_S.
\end{aligned}$$

In the above proof, we have used Eq.(2) and unitarity of $U_p(T)$. It should be stressed that in general, M_μ is not a projective operator. It can be shown

[6] that for an arbitrary quantum state ρ of S, the result μ is observed with probability p_μ evaluated via

$$p_\mu = \mathrm{Tr}\left[\rho M_\mu^\dagger M_\mu\right]. \tag{3}$$

After the measurement, the state of S is transformed into a state given by

$$\rho_\mu = \frac{M_\mu \rho M_\mu^\dagger}{\mathrm{Tr}\left[\rho M_\mu^\dagger M_\mu\right]}. \tag{4}$$

These results are always correct when we start from any indirect measurement model. It has been proven [15] that, inversely, if we have some operators M_μ satisfying Eq.(1), there exists an indirect measurement model with a probe system S' and a measurement interaction between S and S' such that the relations in Eq.(3) and Eq.(4) are reproduced. Hence, we are able to make general arguments on general measurements by considering general operators M_μ satisfying Eq.(1). In mathematics, the set of Hermitian positive semidefinite operators $M_\mu^\dagger M_\mu$ is called a positive operator value measure (POVM). This is because some people call the general measurement POVM measurement.

Here I give comments for measurements in field theory. The localized general measurement operators are expressed as functions of averaged local operators with test functions with compact supports. For example, let us consider arbitrary local operators $O_k(x)$ with $k = 1, 2, \cdots, \infty$ of a field system in one spatial dimension, S. Then the averaged operators are given by

$$\bar{O}_k(R) = \int_{-\infty}^{\infty} \omega_R(x) O_k(x) dx,$$

where $\omega_R(x)$ is a test function with a compact support R. The Hamiltonian of probe interactions depends on time t and those averaged operators as follows.

$$H_p = H_p\left(t, \bar{O}_1(R_1), \bar{O}_2(R_2), \cdots\right).$$

The time evolution operator is given by

$$U_p(T) = \mathrm{T}\exp\left[-i\int_0^T H_p\left(t, \bar{O}_1(R_1), \bar{O}_2(R_2), \cdots\right) dt\right].$$

The general measurements for fields are fixed by giving H_p and the initial state $|S'\rangle$ of the probe system S'. The final state of S, after the ideal measurement of the probe system S' yields the result μ, is expressed by use of the measurement operators M_μ as follows.

$$\operatorname*{Tr}_{S'}\left[P_\mu(S')\left(U_p(T)\left(|S\rangle\langle S|\otimes|S'\rangle\langle S'|\right)U_p^\dagger(T)\right)\right]=M_\mu|S\rangle\langle S|M_\mu^\dagger, \tag{5}$$

where $|S\rangle$ is an arbitrary initial state of S. It is then noticed that M_μ becomes a function of the averaged operators as

$$M_\mu=M_\mu\left(\bar{O}_1(R_1),\bar{O}_2(R_2),\cdots\right).$$

Even if one wants to take a non-separable initial state $|S+S'\rangle$ of S and S' in Eq. (5), the formulation discussed above still works. This is because any $|S+S'\rangle$ is reproduced by acting a unitary operation V on a separable state $|S_I\rangle|S_I'\rangle$:

$$|S+S'\rangle=V|S_I\rangle|S_I'\rangle.$$

Therefore we are able to introduce measurement operators $\tilde{M}_\mu$ satisfying

$$\begin{aligned}&\operatorname*{Tr}_{S'}\left[P_\mu(S')\left(U_p(T)|S+S'\rangle\langle S+S'|U_p^\dagger(T)\right)\right]\\ =\ &\operatorname*{Tr}_{S'}\left[P_\mu(S')\left(U_p(T)V\left(|S_I\rangle\langle S_I|\otimes|S_I'\rangle\langle S_I'|\right)V^\dagger U_p^\dagger(T)\right)\right]\\ =\ &\tilde{M}_\mu|S_I\rangle\langle S_I|\tilde{M}_\mu^\dagger.\end{aligned}$$

The general measurements are generated by probing interactions (expressed by H_p) which are assumed to be switched on in a time interval, $[0,T]$. Effective switching of those couplings may be achieved by various methods in field theory. For example, by applying laser beams to semiconductor devices in quantum optics, energy levels of the devices can be shifted corresponding to the beam strength. This mechanism has been applied to control of photon-counter switching.

LOCC is a setting of quantum communication. Let us consider two parties who share a quantum state of a composite system and want to communicate with each other using the quantum system and classical channels. In the LOCC setting, they are able to perform local operations at each side, including local unitary transformations and local general measurements. The two

parties are also allowed to use classical channels for sending classical information like measurement results. However, they are not allowed to use global quantum operations over the composite system. For example, quantum teleportation [16] is a well-known protocol obtainable by LOCC. It transfers any unknown quantum state to a distant place.

3 Negative Energy Density of Quantum Fields

In this section, we give an overview of negative-energy physics of a 1+1 dimensional free scalar field ϕ. The properties described will be applied to a protocol in the next section. A detailed explanation can be found in [5] and [12]. The equation of motion is

$$\left[\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}\right]\phi(t,x)=0. \tag{6}$$

The general solution of Eq. (6) is written as a sum of left- and right-moving components: $\phi(x,t)=\phi_+\left(x^+\right)+\phi_-\left(x^-\right)$, where $\phi_+\left(x^+\right)$ denotes the left-moving field and $\phi_-\left(x^-\right)$ the right-moving field with light-cone coordinates $x^\pm=t\pm x$. It is remarkable that the quantum interference effect between multi-particle states is able to suppress quantum fluctuation of the field and to yield negative energy density of the field. For example, even though the classical energy flux $\left[\partial_+\phi_+(x^+)\right]^2$ of the left-moving field is nonnegative, the expectation value of the corresponding quantum flux operator $T_{++}\left(x^+\right)=:\partial_+\phi_L(x^+)\partial_+\phi_L(x^+):$ can be negative. Despite the existence of regions with negative energy density, expectation values of the total energy flux $\int_{-\infty}^{\infty}T_{++}(x^+)dx^+$ for an arbitrary state remain nonnegative because the total flux is given by $\int_0^\infty \hbar\omega a_\omega^{+\dagger}a_\omega^+ d\omega$. By taking an arbitrary, monotonically increasing C^1 function $f(x)$of $x\in(-\infty,\infty)$satisfying $f(\pm\infty)=\pm\infty$, the set of mode functions

$$v_\omega(x)=\sqrt{\frac{\hbar}{4\pi\omega}}e^{-i\omega f(x)},\ (\omega\geq 0) \tag{7}$$

is obtained, which can uniquely expand the field. Their orthonormality in terms of the normal product can also be derived straightforwardly. By using mode functions, the left-moving field ϕ_+ is expanded via

$$\phi_+\left(x^+\right) = \int_0^\infty d\omega \left[b_\omega^+ v_\omega(x^+) + b_\omega^{+\dagger} v_\omega^*(x^+)\right].$$

Here $b_\omega^{+\dagger},\ b_\omega^+$ are creation and annihilation operators that satisfy $\left[b_\omega^+,\ b_{\omega'}^{+\dagger}\right] = \delta\left(\omega - \omega'\right)$. We note that the normalized quantum state $|\Phi\rangle$ defined by $b_\omega^+|\Phi\rangle = 0$ is a squeezed state. For $|\Phi\rangle$, the expectation value is evaluated through

$$\langle\Phi|T_{++}\left(x^+\right)|\Phi\rangle = -\frac{\hbar}{24\pi}\left[\frac{\dddot{f}\left(x^+\right)}{\dot{f}\left(x^+\right)} - \frac{3}{2}\left(\frac{\ddot{f}\left(x^+\right)}{\dot{f}\left(x^+\right)}\right)^2\right], \tag{8}$$

where the dot denotes a derivative in terms of x^+[7]. An interesting example of negative energy flux is generated by a monotonically increasing C^1 function $f_\varepsilon(x)$ given by

$$\begin{aligned} f_\varepsilon(x) &= \Theta\left(x_i - x\right)x \\ &\quad + \Theta\left(x_f - x\right)\Theta\left(x - x_i\right)\left[x_i - \frac{1}{\sqrt{\varepsilon}} + \frac{1}{\sqrt{\varepsilon} - \varepsilon\left(x - x_i\right)}\right] \\ &\quad + \Theta\left(x - x_f\right)\left[\frac{\varepsilon}{\left(\sqrt{\varepsilon} - \varepsilon\left(x_f - x_i\right)\right)^2}\left(x - x_f\right) + x_i - \frac{1}{\sqrt{\varepsilon}} + \frac{1}{\sqrt{\varepsilon} - \varepsilon\left(x_f - x_i\right)}\right], \end{aligned}$$

where $x_i \le x_f$, $\Theta(x)$ is a step function and $\varepsilon = \left(\frac{12\pi|E_n|}{\hbar}\right)^2$ is a nonnegative constant. For the squeezed state $|\Phi_{shock}\rangle$ corresponding to $f_\varepsilon(x)$, the left-moving energy flux is estimated by

$$\langle\Phi_{shock}|T_{++}(x^+)|\Phi_{shock}\rangle = -\left|E_n\right|\delta(x^+ - x_i) + \frac{\left|E_n\right|}{1 - \frac{12\pi}{\hbar}\left|E_n\right| l}\delta\left(x^+ - x_f\right), \tag{9}$$

where $l = x_f - x_i (> 0)$. The first term on the right-hand side shows the flux of a shock wave with negative energy $-\left|E_n\right|$. Because $\int_{-\infty}^{\infty}\langle\Phi_{shock}|T_{++}(x)|\Phi_{shock}\rangle dx$ is positive, we obtain the following inequality

$$\frac{\left|E_n\right|^2 l}{\frac{\hbar}{12\pi} - \left|E_n\right| l} \ge 0.$$

Because the numerator is definitely positive, the denominator must be nonnegative, which leads to an uncertainty-relation-type inequality:

$$l = x_f - x_i \leq \frac{\hbar}{12\pi \left|E_n\right|}. \tag{10}$$

This means that negative-energy shock waves cannot be separated infinitely far from positive-energy shock waves. This is because the existence of negative energy is sustained by a quantum correlation effect with positive-energy excitations. If the quantum correlation vanishes completely, negative energy cannot appear in any region because nonnegativity of the Hamiltonian should hold in every local region. Hence, it can be concluded that the negative-energy shock waves form loosely bounded states with the positive-energy shockwaves. Creation of the above loosely bound states of negative- and positive-energy excitations is not peculiar to this example, but rather takes place in any arbitrary system with negative-energy local excitations.

4 Energy Extraction from Local Vacuums by LOCC

In this section, a protocol for energy extraction from local vacuums by LOCC is proposed for a free massless scalar field ϕ in 1+1 dimensions. The system is introduced as a toy model to present a new idea which can be applied to 3+1 dimensional electromagnetic field. We may consider that ϕ corresponds to asymptotic field of QED gauge field in the analogy. Hereinafter, we will refer to this protocol as quantum field energy teleportation (QFET). In canonical quantization, the standard commutation relations are set for the canonical Schrődinger operators as follows.

$$\left[\hat{\phi}\left(x\right),\ \hat{\Pi}\left(x'\right)\right] = i\delta\left(x - x'\right),$$

$$\left[\hat{\phi}(x),\ \hat{\phi}(x')\right] = 0,$$

$$\left[\hat{\Pi}\left(x'\right),\ \hat{\Pi}\left(x'\right)\right] = 0.$$

The energy density operator is written as

$$\hat{\varepsilon}(x) = \frac{1}{2}\left[\hat{\Pi}^2 + \left(\partial_x \hat{\phi}\right)^2\right] - \varepsilon_0,$$

where ε_0 is a constant for subtraction of the vacuum contribution. The Hamiltonian is given by spatial integration of $\hat{\varepsilon}(x)$ as $\hat{H} = \int \hat{\varepsilon}(x)dx$. The vacuum $|0\rangle$ is the eigenstate corresponding to the lowest eigenvalue of $\hat{H}$. By adjusting ε_0, we can set

$$\langle 0|\hat{\varepsilon}\left(x\right)|0\rangle = 0,$$
$$\hat{H}|0\rangle = 0.$$

This choice of ε_0 corresponds to the normal order prescription. The evolution operator of the system is defined by $U(t) = e^{-it\hat{H}}$. Then, using the Schrődinger operators, the canonical Heisenberg operators are calculated as

$$\begin{aligned}\hat{\phi}(t,x) = &\frac{1}{2}\left[\hat{\phi}(x+t) + \hat{\phi}(x-t)\right] \\ &+ \frac{1}{2}\int_{x-t}^{x+t} \hat{\Pi}(y)dy,\end{aligned} \tag{11}$$

$$\begin{aligned}\hat{\Pi}(t,x) = &\frac{1}{2}\left[\hat{\Pi}(x+t) + \hat{\Pi}(x-t)\right] \\ &+ \frac{1}{2}\left[\partial_x\hat{\phi}(x+t) - \partial_x\hat{\phi}(x-t)\right].\end{aligned} \tag{12}$$

Let us consider Alice at $x = x_A$ who excites the ground state of the field by a local measurement, and Bob who stays at $x = x_B$ away from Alice and extracts energy from the field. Then the QFET protocol is composed of the following four phases:

(1) At time $t = 0$, Alice makes a local general measurement defined by operators $M_n\left(A\right)$ satisfying

$$\sum_n M_n^\dagger\left(A\right) M_n\left(A\right) = 1 \tag{13}$$

to the vacuum state $|0\rangle$ and obtains the result n. To perform this measurement, she must, on average, give positive energy E_A to the field. Using this energy, positive-energy wavepackets of the field are generated.

(2) At time $t = t_o$, the wavepackets excited by Alice have already passed by the position of Bob. Assume that Bob fails to catch any energy of the wavepackets at all. Consequently, no energy of ϕ remains around Bob after $t = t_o$.

(3) Alice announces the measurement result n to Bob by classical communication. Bob receives the information at time $t = T(\geq t_o)$.

(4) At $t = T$, Bob performs a unitary operation depending on the value of n defined by

$$U_n\left(B\right) = \exp\left[iga_n \int_{-\infty}^{\infty} p_B(x)\hat{\phi}(x)dx\right], \tag{14}$$

where g is a real constant fixed below, a_n are real constants depending on n and $p_B(x)$ is a function whose support is localized around Bob's location. In this process, positive energy E_B is released on average from ϕ to Bob's apparatus of $U_n\left(B\right)$. In the system of ϕ, wavepackets are generated with negative energy $-E_B$ around Bob's location. Soon afterwards, the wavepackets begin to chase after the positive-energy wavepackets generated by Alice.

The schematics in Figures 1 to 3 describe this QFET protocol with plots of $\langle\varepsilon(x)\rangle = \mathrm{Tr}\left[\rho\hat{\varepsilon}(x)\right]$ as a function of x. A spacetime diagram for protocol events is given in Figure 4. The amount of energy E_A is evaluated by

$$E_A = \sum_n \langle 0| M_n^\dagger\left(A\right)\hat{H}M_n\left(A\right)|0\rangle > 0.$$

After phase (1), the quantum state is transformed into the following state depending on n.

$$|A_n\rangle = \frac{1}{\sqrt{\langle 0|M_n^\dagger\left(A\right)M_n\left(A\right)|0\rangle}}M_n\left(A\right)|0\rangle. \tag{15}$$

The average quantum state after measurement evolves until $t = T$ as follows:

$$\rho(T) = \sum_n U(T)M_n\left(A\right)|0\rangle\langle 0|M_n^\dagger\left(A\right)U^\dagger(T). \tag{16}$$

Soon after phase (4), the average quantum state transforms into the following state:

$$\rho_F = \sum_n U_n(B)\, U(T) M_n(A) |0\rangle\langle 0| M_n^\dagger(A)\, U^\dagger(T) U_n^\dagger(B)\,.$$

In order to evaluate E_B, let us introduce a localized energy operator ϕ around Bob:

$$\hat{H}_B = \int w_B(x)\, \hat{\varepsilon}(x)\, dx,$$

where w_B is a nonnegative window function that satisfies

$$w_B(x) = 1$$

for $x \in (x_B - \epsilon, x_B + \epsilon)$ with a positive constant ϵ and rapidly decreases outside the region $(x_B - \epsilon, x_B + \epsilon)$. Also, we assume that

$$w_B(x)\, p_B(x) = p_B(x)\,.$$

In order to calculate $\mathrm{Tr}\left[\rho_F \hat{H}_B\right]$, recall that the Schrődinger operators in $\hat{\varepsilon}(x)$ are transformed by $U_n(B)$ via

$$U_n^\dagger(B)\, \hat{\phi}(x)\, U_n(B) = \hat{\phi}(x)\,,$$

$$U_n^\dagger(B)\, \hat{\Pi}(x)\, U_n(B) = \hat{\Pi}(x) + g a_n p_B(x)\,.$$

Using these relationships, we obtain

$$U_n^\dagger(B)\, \hat{H}_B U_n(B) = \hat{H}_B + g a_n \hat{O}_B + \frac{g^2}{2} a_n^2 \int p_B(x)^2\, dx,$$

where operator $\hat{O}_B$ is defined by

$$\hat{O}_B = \int p_B(x)\, \hat{\Pi}(x) dx.$$

The localized energy $\langle \hat{H}_B \rangle = \mathrm{Tr}\left[\rho_F \hat{H}_B\right]$ is then given by

$$\langle \hat{H}_B \rangle = \sum_n \langle 0| M_n^\dagger(A) \left(U^\dagger(T) U_n^\dagger(B)\, \hat{H}_B U_n(B)\, U(T) \right) M_n(A) |0\rangle,$$

where

$$
\begin{aligned}
& U^\dagger(T)U_n^\dagger\left(B\right)\hat{H}_B U_n\left(B\right)U(T) \\
= \;& U^\dagger(T)\hat{H}_B U(T) + g a_n \hat{O}_B(T) + \frac{g^2}{2}a_n^2 \int p_B\left(x\right)^2 dx,
\end{aligned} \tag{17}
$$

and $\hat{O}_B(T) = U^\dagger(T)\hat{O}_B U(T)$. It is noted that the above operator commutes with $M_n\left(A\right)$ at time T. This is because the relations

$$
\left[U^\dagger(T)\hat{H}_B U(T),\ M_n(A)\right] = 0, \tag{18}
$$

$$
\left[g a_n \hat{O}_B(T),\ M_n(A)\right] = 0 \tag{19}
$$

hold. Equations (18) and (19) can be verified through Eq.(12) and a relation obtained by differentiation of Eq.(11) such that

$$
\begin{aligned}
\partial_x \hat{\phi}(t,x) = & \frac{1}{2}\left[\partial_x\hat{\phi}(x+t) + \partial_x\hat{\phi}(x-t)\right] \\
& + \frac{1}{2}\left[\hat{\Pi}(x+t) - \hat{\Pi}(x-t)\right].
\end{aligned}
$$

Thus, we are able to obtain a relationship such that

$$
\langle \hat{H}_B \rangle = \sum_n \langle 0| M_n^\dagger\left(A\right) M_n\left(A\right)\left(U^\dagger(T)U_n^\dagger\left(B\right)\hat{H}_B U_n\left(B\right)U(T)\right)|0\rangle. \tag{20}
$$

Substituting Eq.(17) into Eq.(20) yields the following:

$$
\begin{aligned}
\langle \hat{H}_B \rangle = & \langle 0| \left(\sum_n M_n^\dagger\left(A\right) M_n\left(A\right)\right) U^\dagger(T)\hat{H}_B U(T)|0\rangle \\
& + g\langle 0| \left(\sum_n a_n M_n^\dagger\left(A\right) M_n\left(A\right)\right)\hat{O}_B(T)|0\rangle \\
& + \frac{g^2}{2}\int p_B\left(x\right)^2 dx \langle 0| \left(\sum_n a_n^2 M_n^\dagger\left(A\right) M_n\left(A\right)\right)|0\rangle.
\end{aligned}
$$

Let us define Hermitian operators $\hat{D}_A$ and $\tilde{D}_A^2$ as

$$
\hat{D}_A = \sum_n a_n M_n^\dagger\left(A\right) M_n\left(A\right), \tag{21}
$$

$$\tilde{D}_A^2 = \sum_n a_n^2 M_n^\dagger (A)\, M_n (A)\,. \tag{22}$$

Using Eqs.(13), (21) and (22), $\langle \hat{H}_B \rangle$ can be simplified into:

$$\begin{aligned}\langle \hat{H}_B \rangle &= \langle 0| U^\dagger(T) \hat{H}_B U(T) |0\rangle \\ &+ g \langle 0| \hat{D}_A \hat{O}_B(T) |0\rangle \\ &+ \frac{g^2}{2} \int p_B\,(x)^2\, dx \langle 0| \tilde{D}_A^2 |0\rangle.\end{aligned}$$

Because $U(T)|0\rangle = |0\rangle$ and $\langle 0|\hat{H}_B|0\rangle = 0$, the first term on the right-hand side vanishes. Hence, $\langle \hat{H}_B \rangle$ is given by

$$\langle \hat{H}_B \rangle = \frac{1}{2}\xi g^2 + \eta g,$$

where constants ξ and η are defined as

$$\xi = \langle 0| \tilde{D}_A^2 |0\rangle \int p_B(x)^2 dx, \tag{23}$$

$$\eta = \langle 0| \hat{D}_A \hat{O}_B(T) |0\rangle. \tag{24}$$

By fixing the parameter g by

$$g = -\frac{\eta}{\xi},$$

we obtain a negative value for $\langle \hat{H}_B \rangle$ given by

$$\langle \hat{H}_B \rangle = -\frac{\eta^2}{2\xi} < 0. \tag{25}$$

It can be shown explicitly from Eq.(16) that expectation value of energy density is exactly zero right before the operation in phase (4). This is because $\rho(T)$ in Eq. (16) is a local vacuum state (or strictly localized state) in which physical properties around B are the same of those of vacuum. Vanishing of $\mathrm{Tr}\left[\rho(T)\hat{H}_B\right]$ is regarded as an example of general results about statistical independence of separable localized regions (see e.g. [17]). Because $\langle \hat{H}_B \rangle$ becomes negative shortly after phase (4), this field system releases positive energy to Bob's apparatus $U_n\,(B)$. The amount of energy is given by $E_B = -\langle \hat{H}_B \rangle = \eta^2/(2\xi)\,.$ We note that for the quantum state $\rho(T)$ in Eq.(16) many-point functions of the field are equal to those of the vacuum state in

the vicinity of Bob. Consequently, we can regard phase (4) as an energy-extraction process from the local vacuum of ϕ around Bob. It is a typical property of negative energy that negative-energy wavepackets generated by Bob cannot evolve independently of positive-energy wavepackets generated first by Alice. As mentioned in section 3, this is because the existence of negative energy is sustained by the existence of positive energy so as to make the total energy in space nonnegative. Negative energy density is able to emerge only in spatial regions that have finite correlation to other spatial regions with positive energy density. Thus, it is impossible to separate a wavepacket with a fixed negative energy far from wavepackets with positive energy. This observation teaches us that Bob's wavepackets begin to chase after Alice's wavepackets and form a loosely bound state with them after phase (4). At first glance, this statement about loosely bound states might seem irrelevant because the system is in one spatial dimension and both Alice's and Bob's wavepackets maintain their interval while propagating with the same velocity. However, in higher-dimensional field theory, the traveling direction of the first positive-energy wavepackets generally do not have an isotropic distribution and, in particular, there may be a spatial region through which no wavepacket passes. If Bob stays at a point in such a region and makes his local operations, formation of the above loosely bound states becomes a rather nontrivial phenomenon.

5 Example

Although the protocol works for any general measurement by Alice, we present a simple example of a two-valued general measurement. This will allow us to experiment with a similar protocol extended for the electromagnetic field. Let us choose $M_n(A)$ as follows:

$$M_0(A) = \cos\hat{\Phi}_A, \tag{26}$$

$$M_1(A) = \sin\hat{\Phi}_A, \tag{27}$$

where $\hat{\Phi}_A$ is a Hermitian operator given by

$$\hat{\Phi}_A = \frac{\pi}{4} - \int \lambda_A(x)\hat{\Pi}(x)dx,$$

and $\lambda_A(x)$ is a real localized function around Alice's location. The POVM can be constructed by combining the system with a two-state probe system P by a certain interaction. Let us consider an orthonormal state basis $\{|0_P\rangle, |1_P\rangle\}$ of P. Then let us give an interaction Hamiltonian defined by

$$H_p = ig(t)\hat{\Phi}_A \otimes [|1_P\rangle\langle 0_P| - |0_P\rangle\langle 1_P|] \, .$$

Using the time evolution operator $V(t) = \exp\left[-i\int_0^t g(t)dtH_p\right]$, it can be easily proven that the measurement is reproduced at time satisfying $\int_0^t g(t)dt = 1$ by an ideal measurement of an observable $|0_P\rangle\langle 0_P| - |1_P\rangle\langle 1_P|$ for the probe system:

$$M_b \rho M_b^\dagger = \operatorname*{Tr}_P \left[(I \otimes |b_P\rangle\langle b_P|) \, V \left(\rho \otimes |0_P\rangle\langle 0_P|\right) V^\dagger \right] ,$$

where $b = 0, 1$ and ρ is an arbitrary state of the system. In the above protocol setting, it is assumed that switching of the measurement interaction is performed abruptly such that

$$g(t) = \delta(t - 0).$$

To make our argument more concrete, let us choose parameters a_n as $a_n = (-1)^n$. From Eqs.(23) and (24), the following explicit relations are derived:

$$\hat{D}_A = \sin\left(2\int \lambda_A(x)\hat{\Pi}(x)dx\right) , \tag{28}$$

$$\tilde{D}_A^2 = I. \tag{29}$$

From Eq.(15), the measurement by Alice yields post-measurement states depending on n as a sum of two coherent states given by

$$|A_0\rangle = \frac{1}{\sqrt{2}}\left[e^{i\frac{\pi}{4}}|\lambda_A\rangle + e^{-i\frac{\pi}{4}}|-\lambda_A\rangle\right] ,$$

$$|A_1\rangle = \frac{1}{\sqrt{2}}\left[e^{-i\frac{\pi}{4}}|\lambda_A\rangle + e^{i\frac{\pi}{4}}|-\lambda_A\rangle\right] ,$$

where $|\lambda\rangle$ is a coherent state satisfying $\langle\lambda|\hat{\phi}\left(x\right)|\lambda\rangle = \lambda(x)$ and $\langle\lambda|\hat{\Pi}(x)|\lambda\rangle = 0$. For both post-measurement states, the expectational value of the Heisenberg energy density operator takes the value given by

$$\langle A_n|\hat{\varepsilon}(t,x)|A_n\rangle = \frac{1}{2}\left[\left(\partial_x\lambda_A(x-t)\right)^2 + \left(\partial_x\lambda_A(x+t)\right)^2\right]. \tag{30}$$

Here it should be noticed that energy density vanishes outside the compact supports of $\lambda_A(x-t)$ and $\lambda_A(x+t)$ in Eq.(30) because of locality. The first term on the r.h.s. of Eq.(30) describes a right-moving positive-energy wavepacket with light velocity. The second term describes a left-moving wavepacket. The energy input E_A is given by integration of Eq.(30) as

$$E_A = \int_{-\infty}^{\infty} \left(\partial_x\lambda_A(x)\right)^2 dx. \tag{31}$$

At time $t = T$, Bob gets information about n and performs $U_n(B)$ to the state. The amount of energy gain by Bob can be calculated from Eq.(25) as follows. First, ξ is obtained from Eq.(23) by

$$\xi = \int p_B(x)^2 dx. \tag{32}$$

On the basis of Eq.(24) and a relation such that

$$\begin{aligned}\hat{O}_B(T) = &\frac{1}{2}\int dx\left[p_B(x-T) + p_B(x+T)\right]\hat{\Pi}(x) \\ &+\frac{1}{2}\int dx\left[p_B(x-T) - p_B(x+T)\right]\partial_x\hat{\phi}(x),\end{aligned}$$

it is possible to write η as

$$\begin{aligned}\eta = &\int \langle 0|\hat{D}_A\hat{\Pi}(x)|0\rangle\left[p_B(x-T) + p_B(x+T)\right]dx \\ &+\int \langle 0|\hat{D}_A\partial_x\hat{\phi}(x)|0\rangle\left[p_B(x-T) - p_B(x+T)\right]dx.\end{aligned} \tag{33}$$

It is worth noting that the following relation holds from Eq.(28).

$$\begin{aligned}\langle 0|\hat{D}_A &= \frac{1}{2i}\langle 0|\left[\exp\left(2i\int\lambda_A(x)\hat{\Pi}(x)dx\right) - \exp\left(-2i\int\lambda_A(x)\hat{\Pi}(x)dx\right)\right] \\ &= \frac{1}{2i}\left[\langle -2\lambda_A| - \langle 2\lambda_A|\right].\end{aligned}$$

and that

$$\langle 2\lambda_A|0\rangle = \langle -2\lambda_A|0\rangle = \langle 2\lambda_A|0\rangle^*.$$

Let us introduce a distributional function $\Delta(x)$ by

$$\Delta(x) = 2\langle 0|\dot{\phi}(0,x)\dot{\phi}(0,0)|0\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} |k| e^{ikx} dk.$$

$\Delta(x)$ has a delta-functional contribution at $x=0$ and is evaluated as

$$\Delta(x) = -\frac{1}{\pi |x|^2}$$

for $x \neq 0$. By using $\Delta(x)$, we derive the following relationship:

$$\langle 2\lambda_A|\hat{\Pi}(x)|0\rangle = i\langle 2\lambda_A|0\rangle \int_{-\infty}^{\infty} \Delta(x-y)\lambda_A(y)dy.$$

Hence, we obtain the relation:

$$\langle 0|\hat{D}_A\hat{\Pi}(x)|0\rangle = -\langle 2\lambda_A|0\rangle \int_{-\infty}^{\infty} \Delta(x-y)\lambda_A(y)dy. \tag{34}$$

We are also able to show that

$$\begin{aligned} & \int \langle 0|\hat{D}_A\partial_x\hat{\phi}(x)|0\rangle \left[p_B(x-T) - p_B(x+T)\right] dx \\ = \quad & i\langle 2\lambda_A|0\rangle \int \partial_x\lambda_A(x) \left[p_B(x-T) - p_B(x+T)\right] dx = 0. \end{aligned} \tag{35}$$

Here, the last integral vanishes because there is no overlap between $\partial_x\lambda_A(x)$ and $p_B(x\pm T)$. Substituting Eqs.(34) and (35) into Eq.(33) yields

$$\eta = -\langle 2\lambda_A|0\rangle \int dx \int dy \lambda_A(x) \left[\Delta(x-y-T) + \Delta(x-y+T)\right] p_B(y). \tag{36}$$

By substituting Eqs.(32) and (36) into Eq.(25), we obtain the final expression for E_B:

$$E_B = \frac{\left(\langle 2\lambda_A|0\rangle \int dx \int dy \lambda_A(x) \left[\frac{1}{(x-y-T)^2} + \frac{1}{(x-y+T)^2}\right] p_B(y)\right)^2}{2\pi^2 \int p_B(z)^2 dz}. \tag{37}$$

Extension of the protocol to the 3+1 dimensional electromagnetic field is also possible by adopting three-dimensional measurements and unitary operations. The free gauge field A^μ can be expressed in the Coulomb gauge. The gauge fixing condition is given by $A^0 = 0$ and $\mathrm{div}\,\vec{A} = 0$. The two-valued general measurement operators corresponding to Eqs.(26) and (27) are defined by setting

$$\hat{\Phi}_A = \frac{\pi}{4} - \int \vec{\lambda}_A(\vec{x}) \cdot \vec{E}(\vec{x}) d^3x,$$

where $\vec{E}(\vec{x})$ is the electric field and $\vec{\lambda}_A(\vec{x})$ is a three-dimensional vector local function around Alice's position. The local unitary operation of Bob in Eq.(14) is extended as

$$U_n(B) = \exp\left[iga_n \int \vec{p}_B(\vec{x}) \cdot \vec{A}(\vec{x}) d^3x \right],$$

where $\vec{p}_B(\vec{x})$ is a three-dimensional vector localized function around Bob's position and should satisfy the relation

$$\mathrm{div}\,\vec{p}_B = 0,$$

because of residual gauge symmetry of the gauge fixing. A detailed analysis on the electromagnetic field case will be published elsewhere. Experimental checks of the protocol proposed in this paper may be promising in quantum optics, and stimulate future development of new methods of quantum energy transportation.

6 Conclusion

This paper discusses local vacuum states excited by a local general measurement for the vacuum state of a free massless scalar field in 1+1 dimensions. Properties of local vacuum states are the same as those of vacuum as long as we consider the vanishing-energy regions of those states. A protocol is presented that can partially extract the excitation energy from local vacuum states using both information on the measurement result and a quantum apparatus located away from the measurement point, even if the field has, on average, no quantum energy around the apparatus. As an example, the case

of the two-valued general measurements defined in Eqs.(26) and (27) are analyzed in detail. The energy input for the measurement is given by the result in Eq.(31). The extracted energy is calculated using Eq.(37) for measurement-data-dependent unitary operations given by Eq.(14) with $a_n = (-1)^n$ for $n = 0, 1$.

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References

[1] A. Hosoya and A. Carlini, Phys. Rev. D66 (2002) 104011; L. Braunstein and A. K. Pati, Phys. Rev. Lett. 98 (2007) 080502; P. Hayden and J. Preskill, JHEP 0709(2007)120.

[2] E. R. Livine, D. R. Terno, Phys. Rev. D75 (2007) 084001.

[3] B. Reznik, Found.Phys. 33 (2003) 167.

[4] B. Reznik, Phys. Rev. A 71, (2005) 054301.

[5] M. Hotta, Phys.Lett. A372 (2008) 3752.

[6] M. A. Nielsen and I. L. Chuang, "*Quantum Computation and Quantum Information*", Cambridge University Press, Cambridge (2000); M. Hayashi, "*Quantum Information: An Introduction*", Springer-Verlag (2006).

[7] H. Epstein, V. Glaser and A. Jaffe, Nuovo Chimento36(1965)1016. For references related with negative energy density, see a standard textbook, N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge Univ. Press, 1982).

[8] M. Morris and K. Thorne, Am. J. Phys. 56(1988)395; M. Morris, K. Thorne and Y. Yurtsever, Phys. Rev. Lett. 61(1988)1446.

[9] L. H. Ford and T. A. Roman, Phys. Rev. D41(1990)3662; L. H. Ford and T. A. Roman, Phys. Rev. D46(1992)1328.

[10] L. H. Ford, Proc. R. Soc. (London) A346(1978)227; P. C. W. Davies, Phys. Lett. B 113(1982)393.

[11] L. H. Ford, Phys. Rev. D43(1991)3972; L. H. Ford and T. A. Roman, Phys. Rev. D51(1995)4277; L. H. Ford and T. A. Roman, Phys. Rev. D55(1997)2082; M. J. Pfenning and L. H. Ford, Phys. Rev. D57(1998)3489; L. H. Ford, M. J. Pfenning and T. A. Roman, Phys. Rev. D57(1998)4839; C. Fewster and S. Eveson, Phys. Rev. D58(1998)084010.

[12] E. E. Flanagan, Phys. Rev. **D56**, 4922 (1997).

[13] J. Knight, Math. Phys. 2, 459 (1961).

[14] M. Hotta, "Quantum Energy Teleportation by Ground State Entanglement of Spin Chain Systems", arXiv:0803.0348.

[15] M. Ozawa, J. Math. Phys. 25(1984)79.

[16] C. H. Bennett, G. Brassard, C. Cr épeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. 70(1993)1895.

[17] D. Buchhlotz and S. Summers, Phys.Lett. A337, 17 (2005).

Figure Caption

Figure 1: The first schematic diagram of QFET. Alice stays at $x = x_A$ and Bob at $x = x_B$. Alice performs a general measurement to the vacuum state with energy input E_A and obtains the measurement result n. Then, positive-energy wavepackets are generated in the system and escape to spatial infinity at the speed of light. The expectational value of the energy density $\langle \varepsilon(x) \rangle = \mathrm{Tr}\left[\rho\hat{\varepsilon}(x)\right]$ plotted as a function of x.

Figure 2: The second schematic diagram of QFET. After the wavepacket passes through Bob's location, Alice announces to Bob the measurement result n. Bob obtaining n performs local unitary operation $U_n(B)$ depending on n.

Figure 3: The third schematic diagram of QFET. In the process of $U_n(B)$, Bob gets a positive amount of energy from the field, generating negative-energy wavepackets in the field system.

Figure 4: A spacetime diagram of QFET. Entanglement is created between positive-energy wavepackets generated by Alice and negative-energy wavepackets by Bob, which form a loosely bound state.

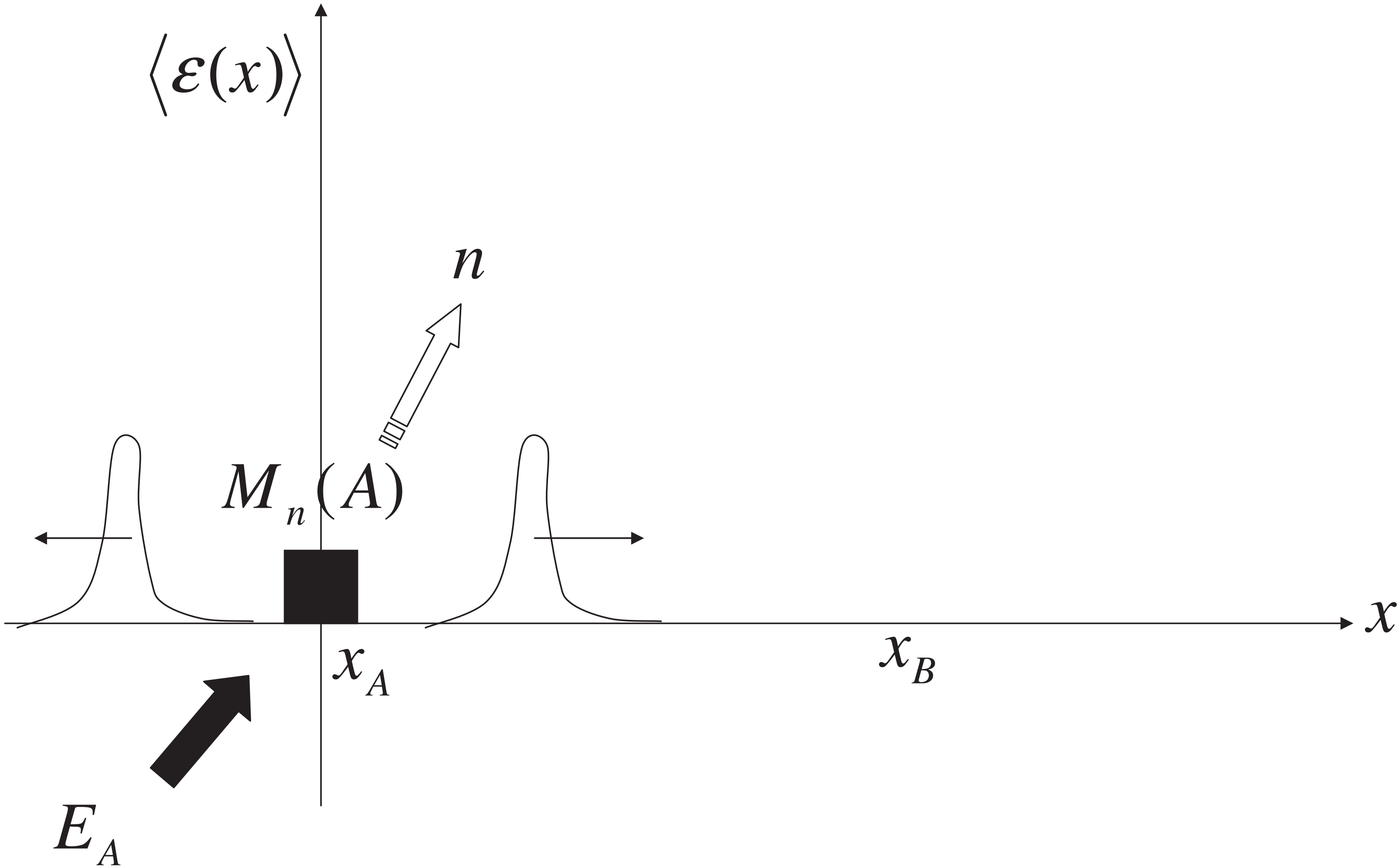


Figure 1

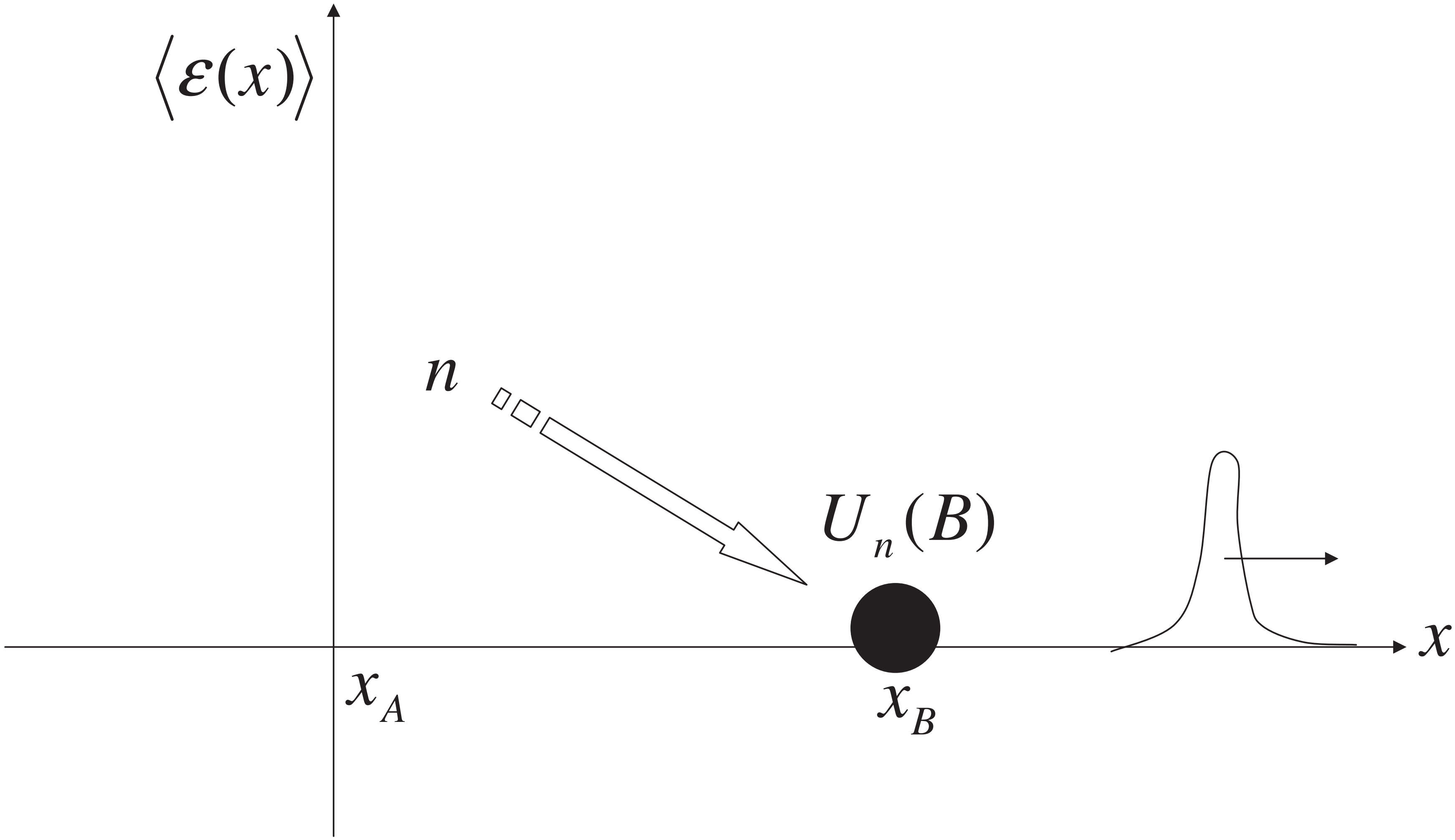


Figure 2

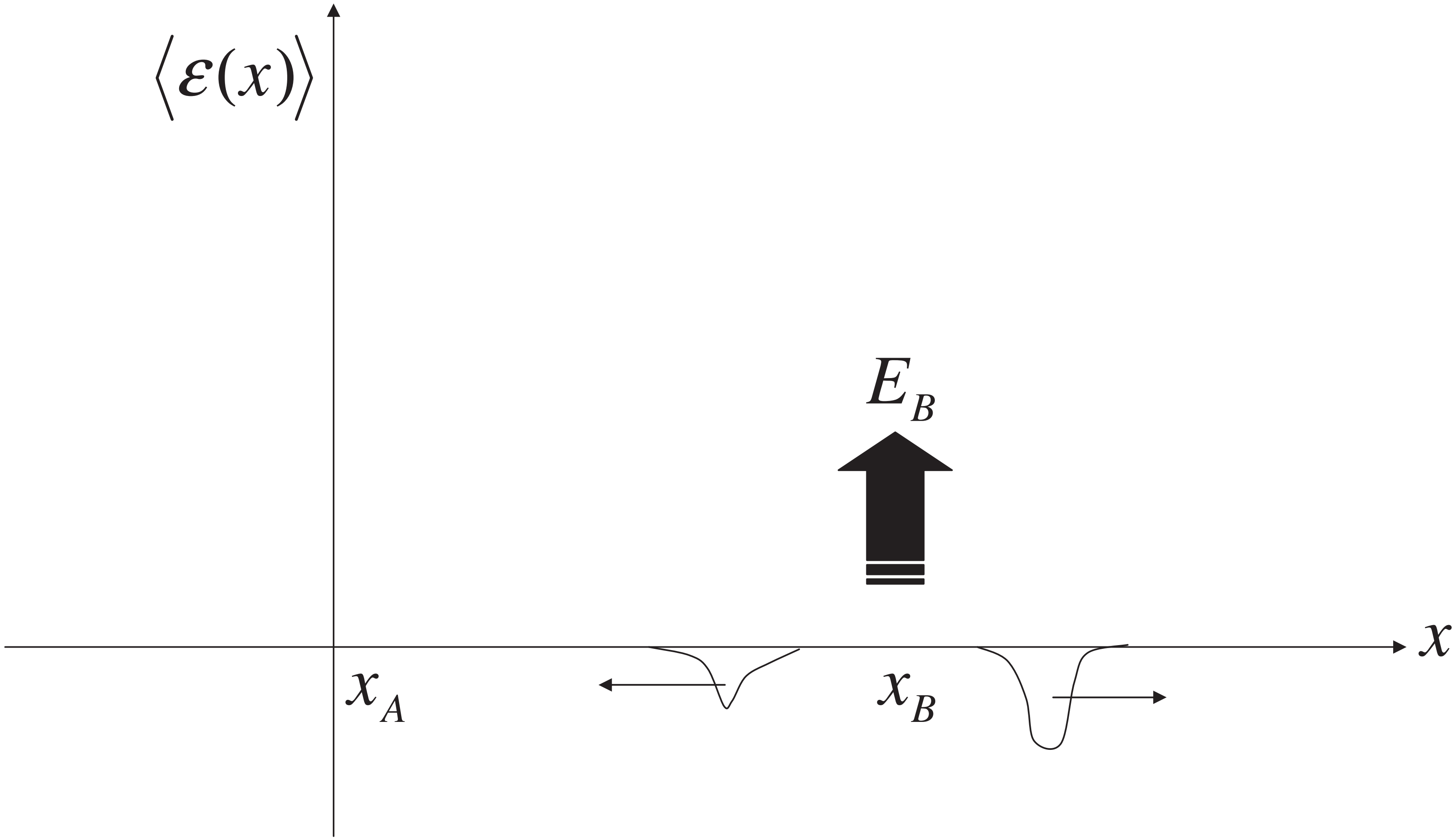
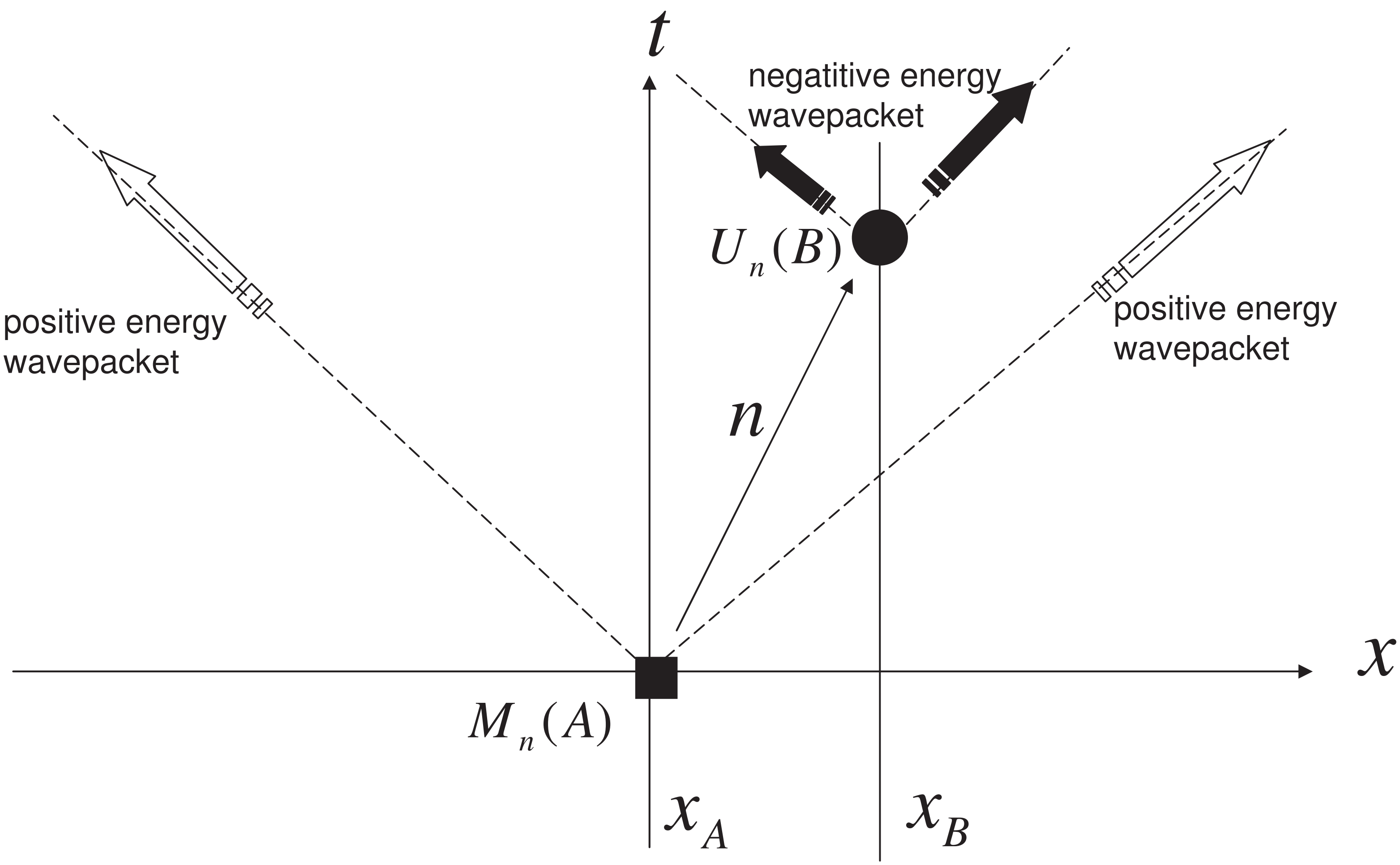


Figure 3



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negative energy
wavepacket
$U_n(B)$
positive energy
wavepacket
positive energy
wavepacket
n
x
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Figure 4